Towards a Formal Grammar of the European Palaeolithic Cave Art

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Abstract. *The figurative component of European Palaeolithic cave art may be divided into fourteen main motifs. A data base consisting of 416 polythematic panels (from two to six different themes) was collected and analysed from a statistical and structural point of view. Factor Analysis and Ascending Hierarchical Classification led to a partition into five classes, one of which is hierarchically dominating (horse, bison, ibex). Using these five classes it was found that only a very small number of the possible combinations have been produced. Moreover, they constitute such a regular pattern that the hypothesis of a random distribution appears highly improbable. A very simple model consisting of a few rewriting rules accounts for the thematic composition of 98% of the Palaeolithic cave art figurative productions. The consistency of the applied structural constraints shows that the inter-thematic associations were governed by semantic choices which remained relatively stable during Upper Palaeolithic in western Europe. A social organisation based on independent, but closely related groups may explain both the stability of the structural principles shown by this work and the regional and chronological diversity of Palaeolithic cave art.*

European Palaeolithic cave art has been extensively studied and many theories about its function and signification have been published. During half a century, all interpretations were based on hunting magic and consequently the images were thought to have been accumulated at random on the cave walls. It is only after the pioneering work by Max Raphaël (1945, 1986) and the comprehensive studies by Laming-Emperaire (1962) and André Leroi-Gourhan (1965) that some coherent patterning began to be recognised in it. For instance, using some very simple statistics, Leroi-Gourhan had attempted to demonstrate that animals and signs were not randomly distributed, as regards their associations and locations in the cave.

Up to now, the implications of this discovery have not been fully examined from a methodological point of view. If rock art conveys a collective thought, it should be structured as any other communication system, in particular as language, and we can take advantage of the recent advances in semiotics and in formal and computational linguistics to study it (Sauvet et al. 1977; Sauvet 1988; Sauvet and Wlodarczyk 1995).

In this paper, we present a general method aimed to demonstrate the existence of common structural features in the paintings and engravings of most French and Spanish Palaeolithic caves. We believe that this method is not specific to the European Palaeolithic cave art and might be applied to many other rock arts.

1. Corpus and data processing

Our first task was to build a database which would be as much representative as possible. Doing so, we have collected 3300 figurative images, coming from 84 caves and rockshelters from France and Spain. For practical reasons, we have limited our study, at the first stage, to *figurative images*, animals and humans which could be clearly identified. We have excluded temporarily all non-figurative images (inappropriately called *signs*), because they are the reason of many problems with classification and typology. Typology is a reducing process which induces a considerable loss of information and is itself a kind of patterning which may obscure the pre-Historic structure we wish to discover. For these reasons, figurative representations constitute a subset of the imagery, particularly suitable for methodological studies, because typology is less determining in that case, and handling of figurative items may facilitate an attempt to correlate the patterns with the underlying semantic networks.

To demonstrate whether these thousands of figures have been put together according to some rules or not, some statistical treatment is necessary, but statistics can only operate on *regularised* data. In other words, data should be processed so as to give them a regular form allowing comparisons.

motif	symbol	number of figures	figures (%)	number of themes	themes (%)
1. Horse	Ho	946	28.7	457	26.4
2. Bison	Bn	730	22.1	307	17.7
3. Ibex	Ib	312	9.5	201	11.6
4. Mammoth	Ma	257	7.8	135	7.8
5. Aurochs	Ox	200	6.1	125	7.2
6. Hind	Hd	239	7.3	122	7.0
7. Stag	St	192	5.8	119	6.9
8. Anthropomorph	An	114	3.4	75	4.3
9. Reindeer	Re	123	3.7	64	3.7
10. Bear	Ur	47	1.4	36	2.1
11. Lion	Li	41	1.2	31	1.8
12. Fish	Po	33	1.0	15	0.9
13. Rhinoceros	Rh	18	0.5	13	0.8
14. Various, Rare	Va	43	1.3	31	1.8
Total		3295	100	1731	100

Table 1. Figurative motifs in European Palaeolithic cave art.

1.1. Graphical categories (motifs). As shown on Table 1, figurative motifs have very different frequencies. Some of them are very frequent: horses and bison, for example, represent together more that 50% of the whole. On the contrary, some other motifs are known only by one or two representations. To limit the number of graphical categories to the most significant ones (from a statistical point of view), we put a threshold at 0.5%. Thirteen motifs stand above this value and the remaining motifs have been gathered in a special category of 'Various rare motifs' (birds, foxes, wolves etc.). Thus, the total number of graphical categories amounts to 14.

1.2. Graphical field. Then, it was necessary to define the semiotic unit or message unit (something comparable to the sentence for a linguist). We have called this unit *graphical field* and it corresponds roughly to a rock panel. In most cases, panels can be easily defined by cracks, flows of calcite or relief features, which were probably perceived as natural limits by the pre-Historic people much in the same way we do so now. Our database contains about one thousand of such panels.

1.3. Thematic reduction. The next step was very important. Once again, in order to facilitate the comparisons, it was necessary to give the data a regular form. This was obtained by giving up the number of times a motif is represented in each panel and taking into account only the occurrence of the motif. Each occurrence of a motif will be called a *theme*, so that a group of ten complete painted bison will be considered as one bison theme in the same way as a small finely engraved bison head. We are aware of the fact that the thematic

reduction as such involves an important loss of information, but we argue that this crude procedure is necessary, in the first approximation, in order to discover the basic rules of the system. The presence or the absence of a theme and the co-occurrences of various themes are obviously fundamental features, far before other aspects such as completeness, size, orientation or technique. Moreover, information which has been temporarily discarded for the sake of this primary analysis might be reintroduced later.

After such a thematic reduction, the 3295 figures of our database are reduced to 1731 themes. This means that a theme is represented, on average, by nearly two items in each of its occurrences. It is noteworthy that the proportions of the 14 motifs are only slightly modified after this reducing operation (Table 1). By order of decreasing frequencies, we find horse, bison and ibex with more than 10% each, followed by mammoth, aurochs, hind and stag. We have distinguished hind and stag, because it was possible owing to sexual dimorphism. This distinction is useful because male and female deer play different roles as shown by their different associations (Sauvet 1988). The next theme is that of humans or more generally speaking anthropomorphs already a relatively rare theme with 4.3%. And finally, reindeer, bear, lion, fish, rhinoceros and the artificial category of 'Various rare'.
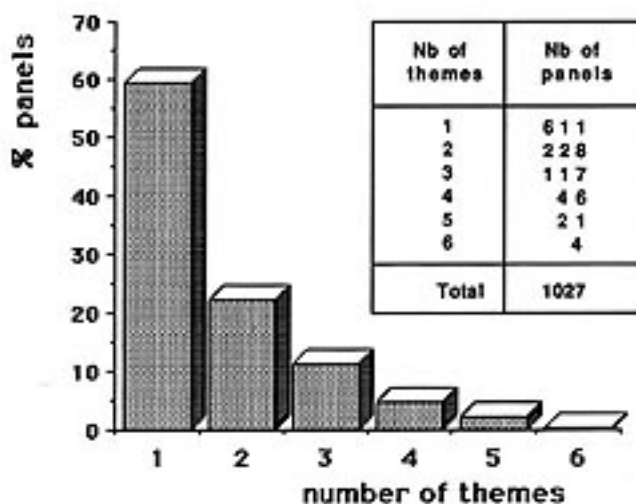

Nb of themes	Nb of panels
1	611
2	228
3	117
4	46
5	21
6	4
Total	1027

Figure 1. Complexity of figurative panels in European Palaeolithic cave art.

2. Statistical analyses

The thematic composition of our 1027 panels is never very complex: 60% of the panels are monothematic and the frequency of the different types of panels decreases rapidly as the complexity increases (Fig. 1). No panel contains more than six different themes of the 14 possible. As it is well known in Communication Theory, the most simple is the most frequent (Mandelbrot 1954).

	Ho	Bn	Ib	St	Ox	Hd	An	Re	Ma	Ur	Li	Rh	Fi	Va
Ho	278	126	78	55	63	41	37	30	26	19	19	7	2	10
Bn	126	186	52	23	13	22	27	30	30	13	4	6	4	9
Ib	78	52	135	38	33	24	16	16	14	8	3	2	3	4
St	55	23	38	97	32	31	7	4	5	3	1	1	1	4
Ox	63	13	33	32	96	16	11	2	8	3	4	2	1	2
Hd	41	22	24	31	16	74	6	4	0	1	1	0	1	2
An	37	27	16	7	11	6	59	4	5	3	3	1	2	3
Re	30	30	16	4	2	4	4	51	8	9	2	0	1	4
Ma	26	30	14	5	8	0	5	10	50	7	0	4	0	1
Ur	19	13	8	3	3	1	3	9	7	29	2	1	0	1
Li	19	4	3	1	4	1	3	2	0	2	21	1	0	3
Rh	7	6	2	1	2	0	1	0	4	1	1	10	0	0
Fi	2	4	3	1	1	1	2	1	0	0	0	0	8	2
Va	10	9	4	4	2	2	3	4	1	1	3	0	2	19

Table 2. Table of co-occurrences of 14 figurative motifs in European Palaeolithic cave art.

As we are mainly interested in the co-occurrences of themes (or *interthematic associations*) that we consider as the basic structuring feature of the system, our corpus is now reduced to the 416 panels which contain two themes or more. From this corpus we can extract a table of co-occurrences (Table 2) that is a symmetrical square table (14 by 14) in which each line represents the associations of a given theme with the other ones and characterises therefore its *thematic distribution*. Computer programs for automatic data processing allow such a table to be submitted to different analyses such as Factor Analysis (Benzécri 1984) and Cluster Analysis (Jambu 1978). Details about the procedures are given in the Appendix.
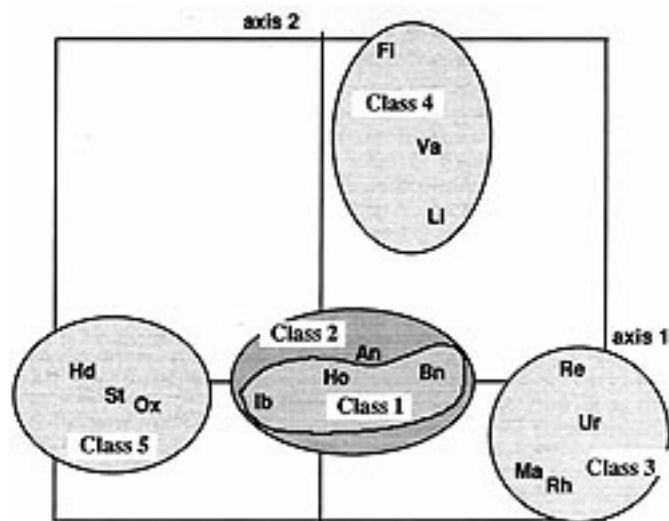
Figure 2. Factor Analysis (Table of co-occurrences of 14 motifs).

2.1. Factor Analysis. According to Factor Analysis, themes that carry the strongest contribution to the definition of axis 1 are hind, stag and ox on the one hand and mammoth, reindeer and bear on the other hand (Fig. 2). This axis might be partly explained by regional *particularity*, hinds and stags being more abundant in Cantabria (Spain) whereas mammoths are mainly abundant in the south-west of France. The second axis separates the mammoth from rare themes: lions, fish and the artificial category 'various'. It is noteworthy that the horse, the bison and the ibex are almost at the centre of gravity of the cloud of points.
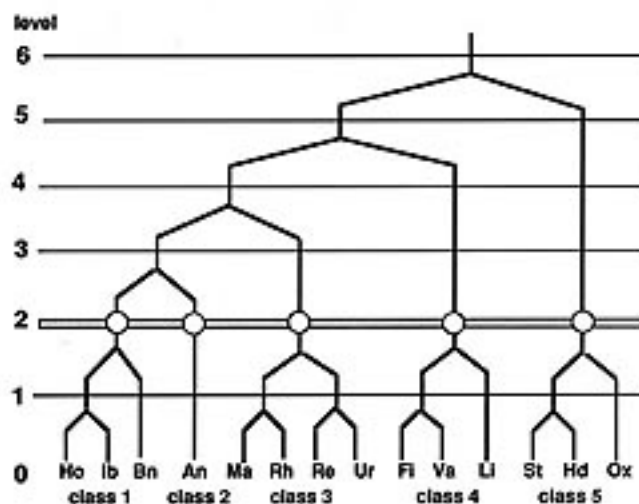

Figure 3. Cluster Analysis (scaled by levels of aggregation).

2.2. Cluster Analysis. Factor Analysis is usually completed by Cluster Analysis. Here the Ascending Hierarchical Classification was carried out on the factor co-ordinates (Fig. 3). The tree-structure which is obtained clearly confirms the clustering which was already apparent on the Factor Analysis. Groups (or classes) are established very soon during the analysis. Thus, if the representation tree is 'cut' at level 2, we get five independent classes:

Horse, ibex and bison	class 1
Anthropomorph	class 2
Mammoth, rhino, reindeer and bear	class 3
Fish, 'various' and lion	class 4
Stag, hind and ox	class 5

When these clusters are reported in the Factor Analysis, we get the pattern shown in Figure 2, which clearly shows the central role played by the horse, the bison and the ibex, the peculiar position of the anthropomorphs near the central group, and the independence of the three peripheral groups.
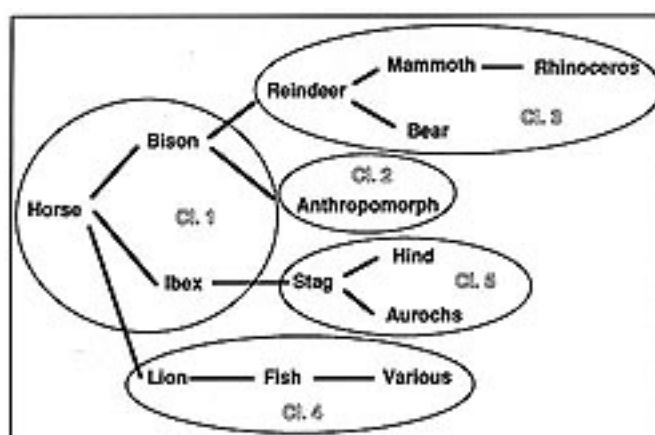

Figure 4. Minimum-cost Spanning Tree (results of the Kruskal algorithm) and classes.

2.3. Kruskal algorithm. The consistency of these five classes was also shown by applying Kruskal algorithm to the table of co-occurrences of themes. This algorithm is able to extract the 'minimum-cost spanning tree' from the complete graph, i.e. the graph in which all the possible links between themes are taken into consideration (Kruskal 1956). This is a way to materialise the strongest links (Fig. 4). The hierarchy of themes so obtained shows that horse is the root of the tree (as expected from its numerical prominence). It is directly attached to bison and ibex, reinforcing thus the reality of class 1. Next, animals of class 3 appear to be linked to class 1 through bison. In the same way, class 5 is attached to class 1 through the ibex, whereas class 4 is directly attached to the horse by the lion. Thus, the five classes derived from Cluster and Factor Analyses appear as branches in the Kruskal tree. Furthermore, Kruskal algorithm shows that the three components of class 1 play a different role with respect to the other classes.

3. Structural model

The statistical study shows that the thematic composition of the panels is strongly structured, but does not provide any explanation for the observed hierarchical pattern. To address this problem, we hypothesised that the structure revealed by statistics reflected a subjacent semantic structure. With this assumption, the five classes could be considered as *semantic* classes and the restricted combinability of themes would be the result of *semantic constraints*. Our objective was to design a set of rules capable of producing the combinations of themes attested in our corpus. These rules would constitute a *formal grammar of Palaeolithic cave art* or, more exactly, a *model* of such a grammar.

3.1. Restricted combinability. The existence of constraints is easily shown by the analysis of the different types of combinations of themes (up to six themes per panel). In fact, the system appears extraordinarily constrained, since only a very small number of combinations (162) have been produced among a very large number of possibilities (6451). As our corpus contains 416 panels, this means that some types are repeated many times (e.g. horse-bison: 40 times) while others have never been produced.
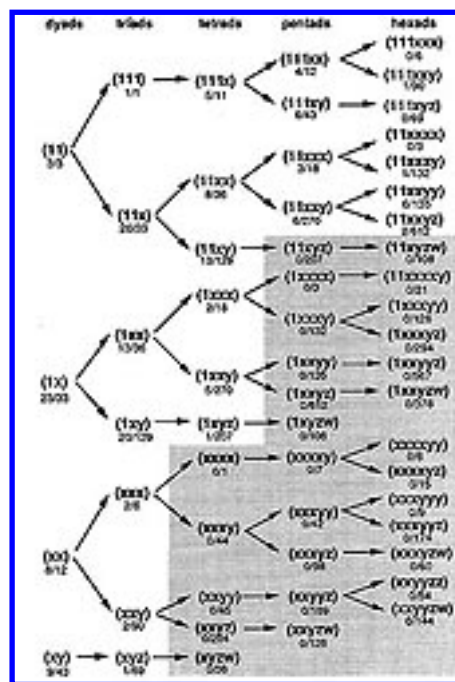

Figure 5. Class composition of panels (number of occurrences / number of theoretical combinations).

To examine the role played by the five classes, we decided to retain only the information on the *class structure* and each theme was replaced by the class to which it belongs. The results are presented in Figure 5 where the elements of class 1 are identified by the number 1, while the elements of the other classes are simply labelled anonymously by letters *w*, *x*, *y* or *z*. The different types of panels are presented in order of increasing complexity and deduced from each other by successive addition of elements. The advantage of this convention is to show that the existing structures constitute a small subset of the enormous

number of possibilities. It is noteworthy that none of the combinations in the grey area is attested in our corpus and that all the existing panels are concentrated in the upper left corner and may be derived recursively. Thus, the thematic constitution of the panels is clearly not random, but governed by some *selection rules*.

We observe that the number of existing panels with respect to the possible combinations decreased drastically with the number of elements of class 1. For instance, all existing tetrads are built from dyads containing at least one element of class 1, and all existing pentads and hexads are issued from a dyad formed by two elements of class 1. This strongly confirms the prominent role of this class.

Unfortunately, we are not able to express the selection rules positively. We can only observe their effect which is to limit the combinability. Thus, the selection rules appear to us as *constraints* which disallow some combinations.

3. 2. Formal grammars. At this point, it is easy to write a program for the computer in a language such as PROLOG which is one of the most used languages in Computational Linguistics (Colmerauer 1975; Pereira and Warren 1980). It would be too long to describe in a few pages how the rules are written and how they work. However, the principle may be shown schematically.

Unconstrained Grammar	Generated combin.	Existing combin.
dyad --> te + te	91	45
triad --> dyad + te	364	59
tetrad --> triad + te	1001	34
pentad --> tetrad + te	2002	20
hexad --> pentad + te	3003	4
	---------	---------
te --> Ho; Bn; Ib; An; Ma; Rh; Re; Ur; Fi; Va; Li; St; Hd; Ox	6461	162

Table 3. Unconstrained grammar producing all combinations up to six themes.

Table 3 presents the skeleton of a Prolog program which defines a dyad as the sum of two terminals, a triad as a dyad + a terminal, and so on, the terminals being the 14 figurative motifs. Of course, this is just a definition of panels. Running this program will generate all the possible combinations. This is an *unconstrained grammar*.

Now, it is possible to add the structural constraints we have seen previously to the rules defining the different types of panels. For instance, 'triad constraints' state that three

elements may not belong to three different classes *X, Y, Z*. 'Tetrad constraints' add the rule that at least one element of class 1 must be present. 'Pentad constraints' and 'hexad constraints' set that at least two elements of class 1 must be present. Indeed, an information concerning the class of each motif should be added to the definition of terminals. This very simple grammar consisting of five *rewriting rules* and a few constraints produces four times less combinations than the unconstrained grammar. Yet 98% of the combinations attested in the caves are produced (Table 4).

Formal Grammar with structural constraints	Generated combin.	Existing combin.
dyad --> te + te	91	45
triad --> dyad + te + *triad-constraints*	295	58
tetrad --> triad + te + *tetrad-constraints*	464	33
pentad --> tetrad + te + *pentad-constraints*	343	20
hexad --> pentad + te + *pentad-constraints*	366	2
	----------	----------
te --> Ho(1); Bn(1); Ib(1); An(2); Ma(3); Rh(3); Re(3); Ur(3); Fi(4); Va(4); Li(4); St(5); Hd(5); Ox(5)	1559	158

Table 4. Formal grammar with structural constraints.

The grammar is no longer deterministic, but probabilistic. This is not a methodological problem since the few combinations that are not accounted for by the model may result from errors in reading of the panels. What is surprising, owing to the simplicity of the formal model we used, is that so few 'abnormal' panels are present.

4. The cave of Chauvet-Pont-d'Arc

We cannot omit to mention the extraordinary discovery of the cave of Chauvet-Pont-d'Arc. The impressive quality and aesthetic force of its numerous paintings and engravings, the richness of innovative techniques make the cave exceptional from many points of view. The most extraordinary was the recent dating around 30 000 BP which makes it the oldest painted cave. The cave will be very useful in the future as a complementary corpus, but nowadays only general comments can be made from the preliminary report recently published (Chauvet et al. 1995).

4.1. Animals. Among the originalities of the cave we find the frequency of some figurative themes. The most abundant is the rhinoceros followed by the lion. Horses and bison are far behind. It is true that the number of rhinos in the cave is twice the total number of these animals represented in other caves. This has been pointed out as a contradiction with Leroi-Gourhan's model. But it is also interesting to note that almost all 300 figures of the cave belong to the 13 main categories we have established from the statistical analysis of dozens of other caves. Only three representations fall into the category of the rare: they are a bird (an owl), a panther and a possible hyena. The originality of Chauvet-Pont-d'Arc is only in the frequencies of representations. The fauna depicted in the cave is *qualitatively* normal, with a quantitative deviation from statistics, which is not a unique case. Let us recall the 18 hinds of Covalanas (on a total of 21 animals) or the 150 mammoths of Rouffignac.

Some pre-Historic social groups, for some reasons, may have developed *locally* and *temporarily* distinctive semantic domains. This does not conflict with the idea of a widespread common 'system of visual communication' favouring inter-group relations which were necessary for many reasons (flint supply, mating networks, seasonal hunting etc.). On the other hand, archaeology has largely demonstrated the existence of such relations through the circulation of flint, shells and portable art. The circulation of ideas concerning cave art probably followed the same routes. This is indirectly proven by the distribution of certain stylistic conventions and some specific 'signs' over wide areas.
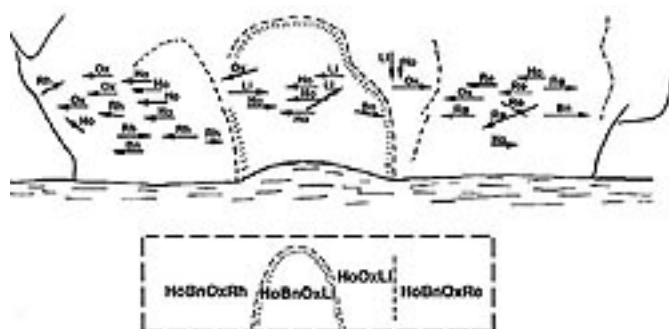

Figure 6. Thematic composition in the Cave of Chauvet-Pont d'Arc (Panel of the Horses).

4.2. Panel construction. The second point to consider at Chauvet-Pont-d'Arc is the construction of the panels. Two of them are particularly interesting because of their amplitude and complexity. The first one has been called '*Panel of the Horses*'. It was executed as a triptych: two lateral panels surrounding a richly decorated niche. The three groups contain horses and at least one aurochs and one bison. In each case, the bison is marginal (in the lower part of the panel or extending out of the niche). A fourth species is represented in each panel: reindeer at right, lions in the middle, and rhinos at left. So the whole set of figures is composed of three tetrads of the same general structure: horse-bison-aurochs + *X*. It is noteworthy that these tetrads are produced by our formal grammar and that the tetrad horse-bison-aurochs-rhino is attested in the terminal gallery of Font-de-Gaume.
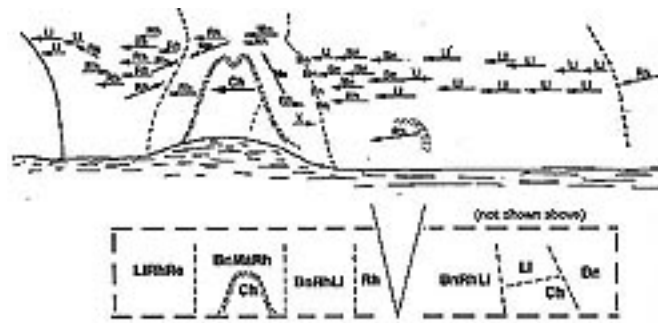
Figure 7. Thematic composition in the Salle Terminale of Chauvet-Pont d'Arc .

In the vast panel of the Salle Terminale, the horse theme is quantitatively rare, but is located in places of special significance: an alcove majestically occupied by a horse alone, and a narrow recess from which a horse seems to be emerging. The niche is surrounded by mammoths and bison, and the left side of the niche features mainly rhinos with a few lions, while the right side contrasts with a large group of lions with only two rhinos. The general impression is that of a central composition horse-bison-mammoth flanked on both sides by rhinos and lions. Although quantitatively dominant and masterfully executed, rhinos and lions are *literally* marginal in the organisation of the panel which is based on a central recess as in the preceding example.

In our opinion, this extraordinary cave is not original in the sense of a short-lived masterpiece, but is a *prototype* of all the other caves that have been decorated by generations of Palaeolithic hunters during millennia. Chauvet-Pont-d'Arc already contains the structural principles which are underlying in all subsequent assemblages of figurative themes.

Additionally, such a complex pattern shows that the thematic composition of a panel is only a part of its meanings. The figurative syntax and the morphological features of the individual figures are also very important.

5. Conclusions

The conclusions of this work may be summarised in a few statements:

1) the number of figurative motifs in European Palaeolithic cave art is small. Some themes like horse, bison and ibex are true leitmotivs.

2) the known associations of figurative themes represent a very small part of the possible combinations;

3) These features may be accounted for by a formal grammar consisting of a small

number of rules. This very simple and coherent model accounts for the thematic composition of 98% of the Palaeolithic cave art productions and constitutes a 'formal grammar model'.

We must emphasise that we are probably dealing with semantic rules reflecting a collective thought in a given domain. Of course, in our opinion, the word 'grammar' does not imply any direct relation with language. It is also important to stress that we do not take the model for granted but as heuristically helping us to bring the regularities to light.

The above conclusions are mere statements of facts which need to be interpreted and discussed in terms of their archaeological meanings. The fact that a small number of coherent and recursive rules may account for the whole of the corpus implies that the same rules have been in use during millennia in the very large area featuring Palaeolithic cave art. Soon after Leroi-Gourhan's work and up to now, many prehistorians have expressed their doubt that the same religious system could last so long. They think that more or less independent *diachronic and synchronic systems* should have occurred. As the matter of fact, during the Upper Palaeolithic period, we see many changes in lithic and bone equipment, in hunting strategies, and probably also in social relationships between groups, but we have no definitive proof that their religious thought has been deeply affected by these adaptive changes.

On the contrary, we may consider that the present work regarding the associations of figurative themes in cave art is a strong argument in favour of a certain stability. In the actual state of this work, we have no reason to suspect the existence of several sub-structures in our database. In other words, the spatial and temporal differences that have been observed by others (and by ourselves) are confined to a formal level. Technical, morphological and stylistic differences do not affect the structure. Indeed, diversity exists and has an important role to play *within* the system, but diversity does not imply that we are dealing with different systems (Conkey 1989).

In this work, we wished only to answer an important and disputed question concerning the existence of a common structure in Palaeolithic cave art. To address this problem, we developed a methodology which led us to use very reduced data (themes instead of individual figures).

As cave art revealed a semantic structure that has remained surprisingly stable through time and space, we think that a similar question concerning syntactic patterns is now worthwhile to be addressed. At this stage, the individual figures should be reintroduced with their morphological attributes such as size, orientation, degree of completeness, attitudes and technique, and their spatial distribution and associations with signs should also be taken into account.

The task is in progress but is difficult for two reasons. First, some of the necessary

information is missing in the literature and must be gathered by long and patient field work. Secondly, new methods able to integrate the new information must be developed. We anticipate that a more sophisticated analysis taking into account technical and morphological characteristics may lead us to discover *diachronic and synchronic variations* that could be correlated with the archaeological contexts.

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APPENDIX

Data analysis was carried out using algorithms developed by Benzecri and his school (Benzecri 1984; Jambu 1978). *Correspondence Analysis* (CA) is a factor analysis which makes use of χ^2 metrics and allows a symmetrical treatment of the objects and the variables which makes possible the simultaneous projection of both on the factorial planes. The CA output indicates, for each factor and each element (variable or object), the contribution of the element to the definition of the factor and the contribution of the factor to the description of the element. The quality of the description in the first projection planes is measured by the weight of the axes (fraction of the total inertia of the cloud of points). For a comparison with other types of factor analyses and applications in archaeology, see Djindjian (1991).

In the present work, the original data were collected as a simple logical table which was a list of objects (the 416 polythematic panels) described by 14 qualitative variables (presence or absence of the 14 motifs respectively coded by 1 or 0). The list was transformed in a symmetrical 14×14 matrix (called 'table of co-occurrences') in which each cell (i, j) represents the number of times motif i was found associated with motif j. In the diagonal, the cells (i, i) contain the number of occurrences of motif i. In the CA shown in Figure 2, the weights of axis 1 and axis 2 are respectively 20.1% and 13.4% of the total inertia.

The partition of the motifs into classes was achieved by *Cluster Analysis* using algorithms also developed by Benzecri's school (Jambu 1978). Ascending Hierarchical Classification (AHC) was chosen and the aggregation criterion was by the highest value of the centred second order moment of a partitioning. It was found particularly useful to take as variables the co-ordinates of the motifs on the factor axes previously obtained by CA. This procedure facilitates the interpretation of the factor analysis by allowing a direct superimposition of the clusters on the projection planes (as shown in Fig. 2).

The computer programs used in this work are based on algorithms written in Fortran language and published by Benzecri and his school (for CA, see Benzecri 1984, vol. 2 and, for AHC, Jambu 1978, vol. 2). We have implemented these programs on Macintosh computers and designed a special interface which makes use of the facilities offered by usual spreadsheets and word processing software for input and output data handling.

The 'optimal tree algorithm' often referred to in the literature as the 'minimum spanning tree algorithm' is based on an algorithm initially proposed by J. B. Kruskal. A Fortran sub-routine may be found in Pelletier (1971).

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dyads	triads	tetrads	pentads	hexads

(111)
1/1

(111x)
5/11

(111xx)
4/12

(111xxx)
0/6

(111xxy)
1/90

(111xy)
6/43

(111xyz)
0/69

(11)
3/3

(11x)
20/33

(11xx)
8/36

(11xxx)
3/18

(11xxxx)
0/3

(11xxxy)
1/132

(11xxy)
6/270

(11xxyy)
0/135

(11xxyz)
2/612

(11xy)
13/129

(11xyz)
0/207

(11xyzw)
0/108

(1xxx)
2/18

(1xxxx)
0/3

(11xxxxy)
0/21

(1xxxy)
0/132

(1xxxyy)
0/126

(1xxxyz)
0/294

(1xx)
13/36

(1xxy)
5/270

(1xxyy)
0/135

(1xxyyz)
0/567

(1xxyz)
0/612

(1xxyzw)
0/378

(1x)
23/33

(1xy)
20/129

(1xyz)
1/207

(1xyzw)
0/108

(xxxx)
0/1

(xxxxy)
0/7

(xxxxyy)
0/6

(xxxxyz)
0/15

(xxx)
2/6

(xxxy)
0/44

(xxxyy)
0/42

(xxxyyy)
0/9

(xxxyyz)
0/174

(xxxyz)
0/98

(xxxyzw)
0/60

(xx)
8/12

(xxyy)
0/45

(xxyyz)
0/189

(xxyyzz)
0/54

(xxyyzw)
0/144

(xxy)
2/90

(xxyz)

(xxyzw)

(xxy)
2/90

(xxyz) $\longrightarrow$ (xxyzw)
0/204   0/126

0/144

(xy) $\longrightarrow$ (xyz) $\longrightarrow$ (xyzw)
9/43   1/69   0/36